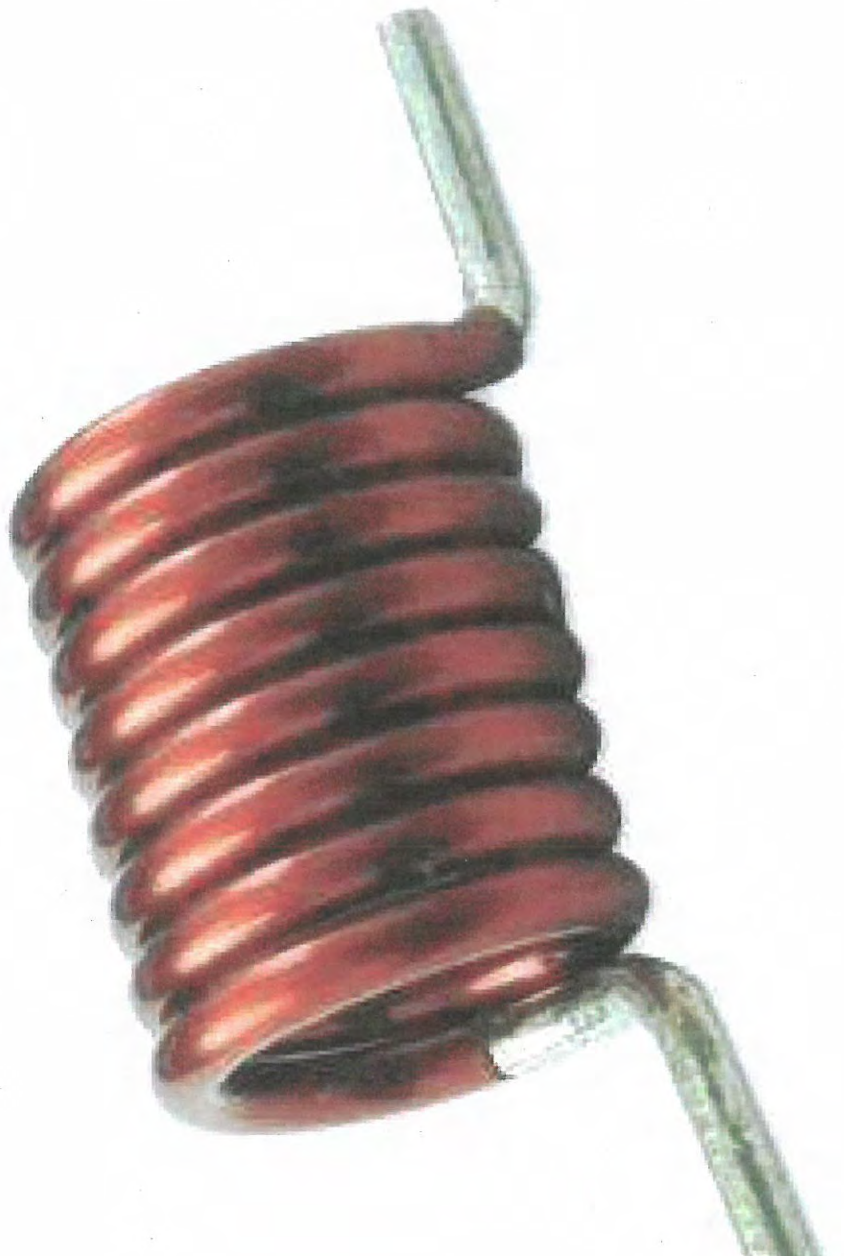
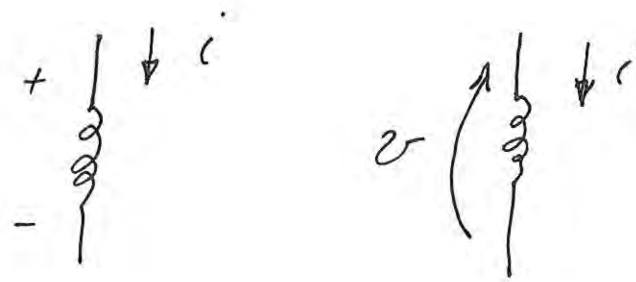


INDUCTOR

 SYMBOLIC REP



PHYSICAL PROPERTY: INDUCTANCE

INDUCTANCE L MEASURED IN HENRYS (H)

$$H = \frac{Wb}{A} = \frac{V \cdot s}{A} = \frac{V}{A/s}$$

V ← VOLTAGE

A/s ← CURRENT/s

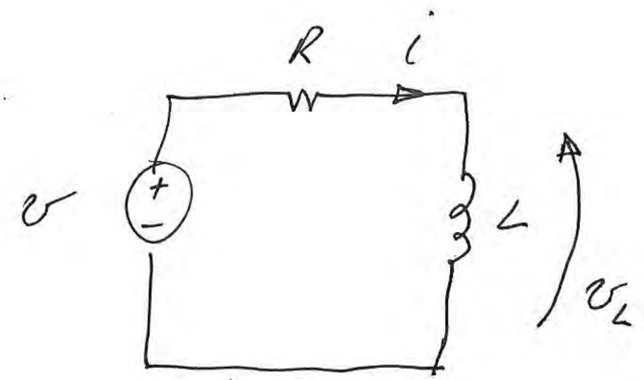
CURRENT-VOLTAGE RELATIONSHIP

$$v = L\frac{di}{dt}$$

v: VOLTS

L: $\frac{\text{VOLTS}}{\text{AMPS/S}}$

$\frac{di}{dt}$: $\frac{\text{AMPS}}{\text{S}}$



$$v_L = L \frac{di}{dt}$$

IF v IS DC, WHAT IS i ?

FOR DC, AN INDUCTOR APPEARS AS A SHORT

$$v(t) = L\frac{di(t)}{dt}$$

$$v(t)dt = L\,di(t)$$

$$v(x)dx = L\,di(x)$$

$$\int_{t_0}^{t} v(x)dx = L\int_{t_0}^{t} di(x)$$

$$= L\left[\,i(t) - i(t_0)\,\right]$$

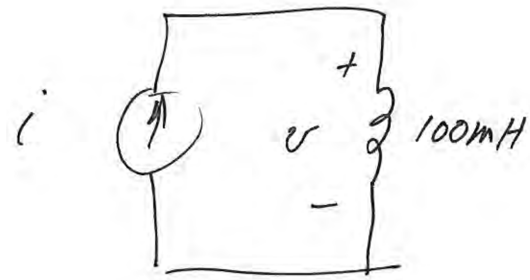
$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(x)dx + i(t_0)$$

PHYSICAL INTERPRETATION:

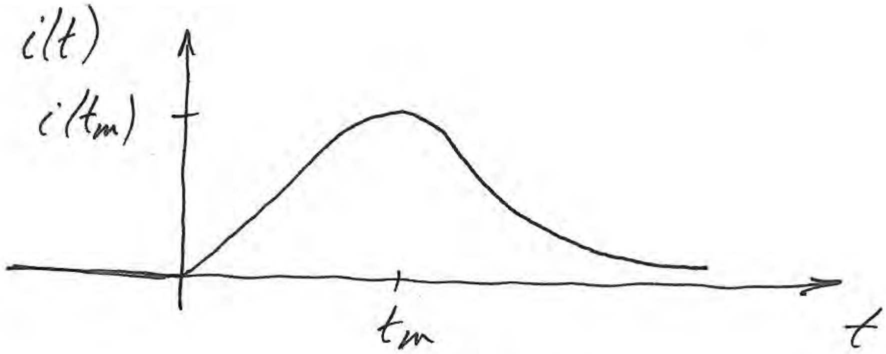
CURRENT THROUGH INDUCTOR

CANNOT CHANGE INSTANTANEOUSLY

EX 6.1 PG 177



$$i = \begin{cases} 0 ; & t < 0 \\ 10te^{-5t}\ A ; & t > 0 \end{cases}$$



$$\frac{di}{dt} = 0 : \quad 10e^{-5t} + 10t(-5)e^{-5t} = 0$$

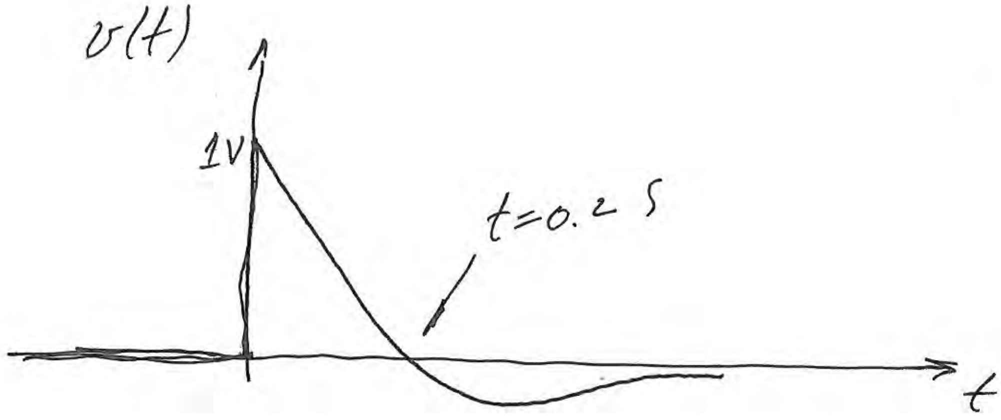
$$1 - 5t = 0$$

$$t_m = 1/5 = 0.2 \text{ SEC}$$

$$i(t_m) = 10(0.2)e^{-5(0.2)} = 2e^{-1} = 0.736 \text{ A}$$

$$v(t) = L\frac{di}{dt} = 0.1 \times 10e^{-5t}(1-5t)$$

$$v(t) = (1-5t)e^{-5t} \text{ V} \quad ; t > 0$$



CURRENT CANNOT CHANGE INSTANTANEOUSLY

VOLTAGE <u>CAN</u> CHANGE INSTANTANEOUSLY

$$i(t) = 10te^{-5t} \text{ A}$$

WHAT ARE UNITS OF "5" ?

WHAT ARE UNITS OF "10" ?

POWER & ENERGY

$$p = v\,i \qquad \text{POWER}$$

(p: WATTS; v: VOLTS; i: AMPS)

BUT $v = L\dfrac{di}{dt}$

$$p = L\,i\,\frac{di}{dt}$$

$$p(t) = L\,i(t)\,\frac{di(t)}{dt}$$

$$p(x)\,dx = L\,i(x)\,di(x)$$

$$\int_{t_0}^{t} p(x)\,dx = L\int_{t_0}^{t} i(x)\,di(x)$$

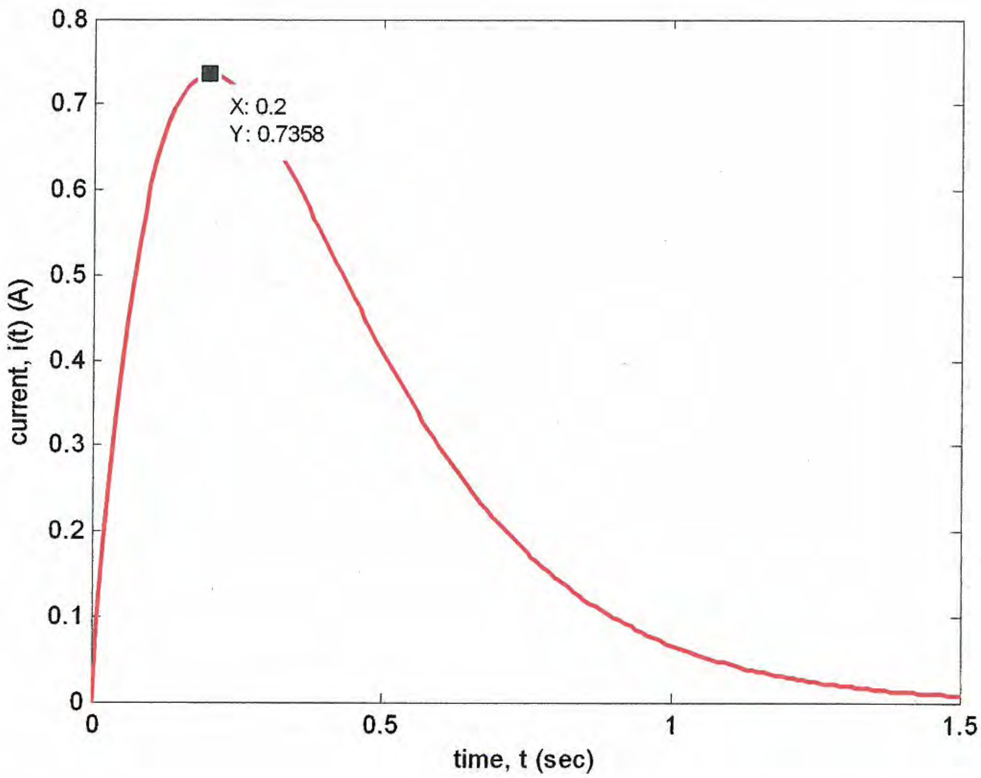
$$w(t) - w(t_0) = \frac{L}{2}\left[\,i^2(t) - i^2(t_0)\,\right] \qquad \text{ENERGY}$$

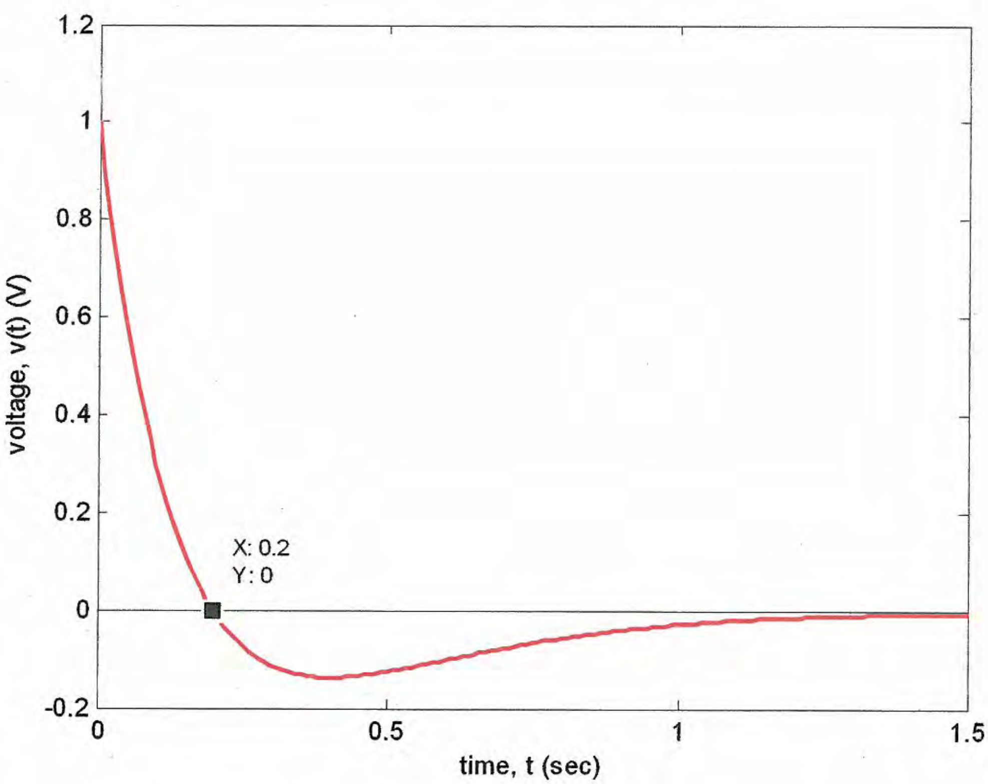
SAY INITIAL CURRENT, $i(t_0) = 0$

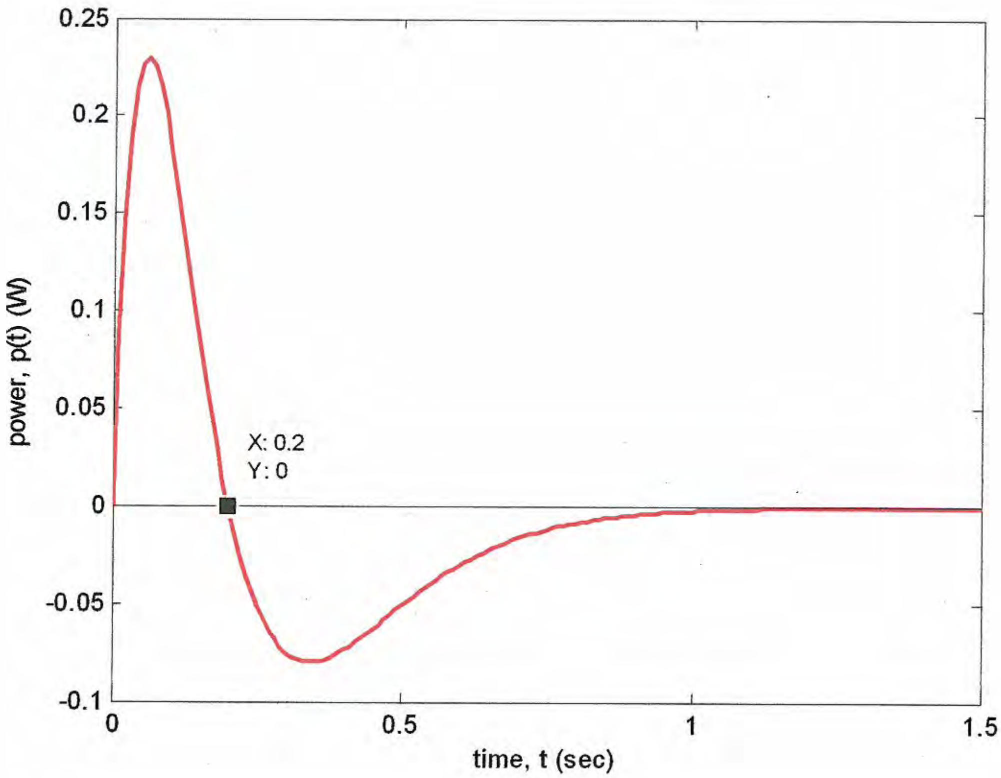
$\Rightarrow$ INITIAL ENERGY $\omega(t_0) = 0$

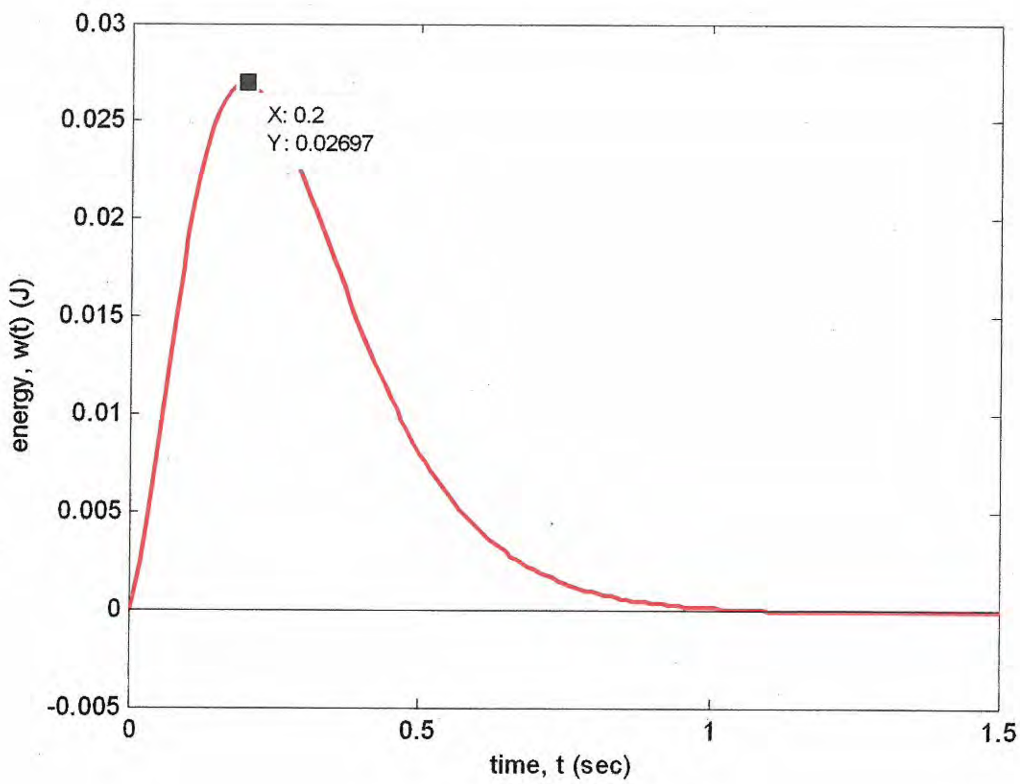
$$\boxed{\omega(t) = \frac{1}{2} L i^2(t)}$$

INSTANTANEOUS ENERGY IN INDUCTOR


0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
current, i(t) (A)
X: 0.2
Y: 0.7358
0
0.5
1
1.5
time, t (sec)


1.2
1
0.8
0.6
0.4
0.2
0
-0.2
voltage, v(t) (V)
X: 0.2
Y: 0
0
0.5
1
1.5
time, t (sec)


0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
power, p(t) (W)
X: 0.2
Y: 0
0
0.5
1
1.5
time, t (sec)


0.03
0.025
0.02
0.015
0.01
0.005
0
-0.005
energy, w(t) (J)
X: 0.2
Y: 0.02697
0
0.5
1
1.5
time, t (sec)

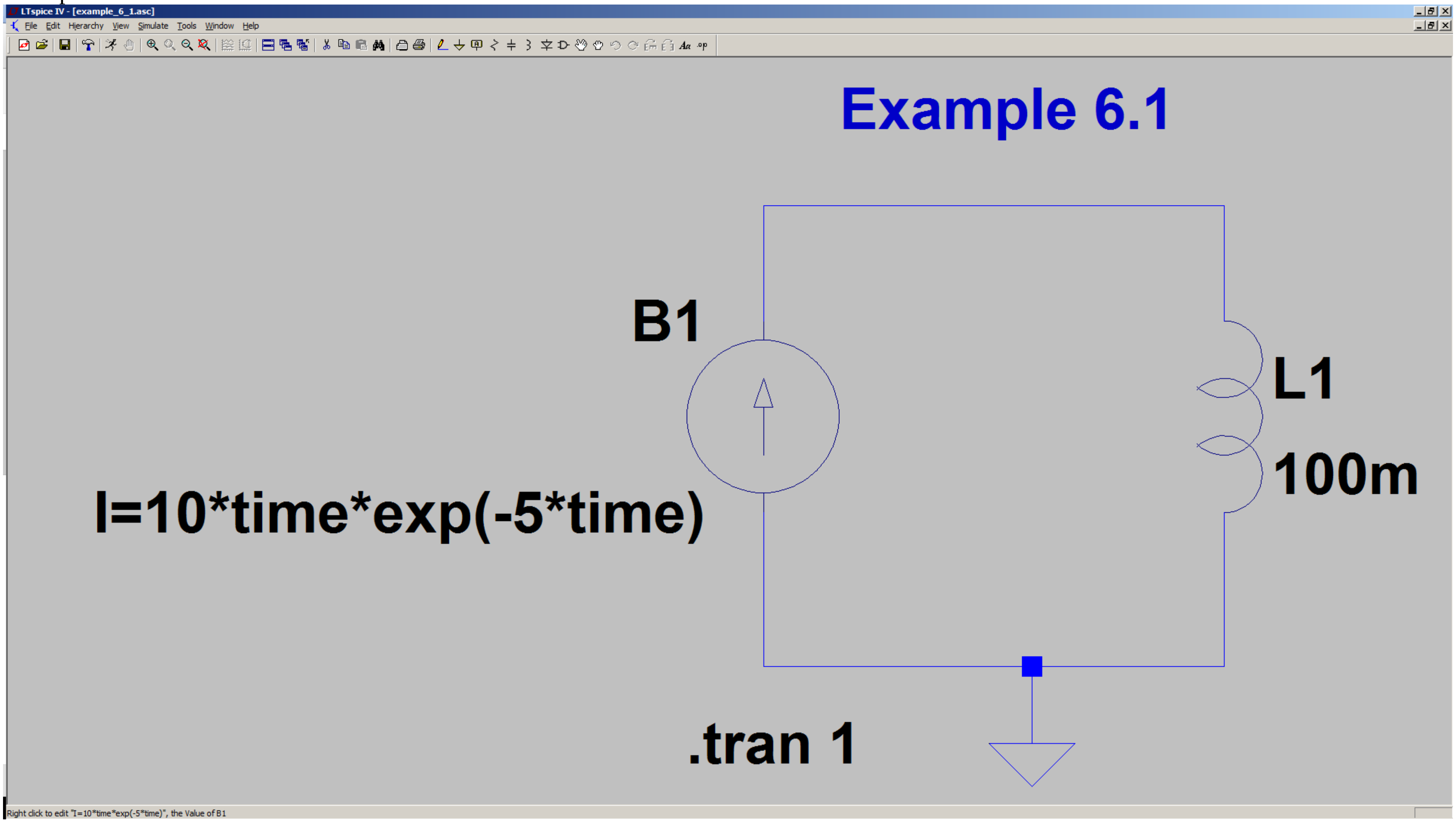
```
% example_6_1
% Nilsson & Riedel 10th ed.
% 01/03/15 D D Duncan
%
dt = 0.01;
t = 0:dt:1.5;
i = 10*t.*exp(-5*t);% units of amperes
v = (1 - 5*t).*exp(-5*t);% units of volts
p = i.*v;
w = cumsum(p)*dt;
figure(1);plot(t,i,'r-');
xlabel('time, t (sec)');ylabel('current, i(t) (A)');
figure(2);plot(t,v,'r-',[t(1) t(end)],[0 0],'k-');
xlabel('time, t (sec)');ylabel('voltage, v(t) (V)');
figure(3);plot(t,p,'r-',[t(1) t(end)],[0 0],'k-');
xlabel('time, t (sec)');ylabel('power, p(t) (W)');
figure(4);plot(t,w,'r-',[t(1) t(end)],[0 0],'k-');
xlabel('time, t (sec)');ylabel('energy, w(t) (J)');
```

$v = iR$ RESISTOR

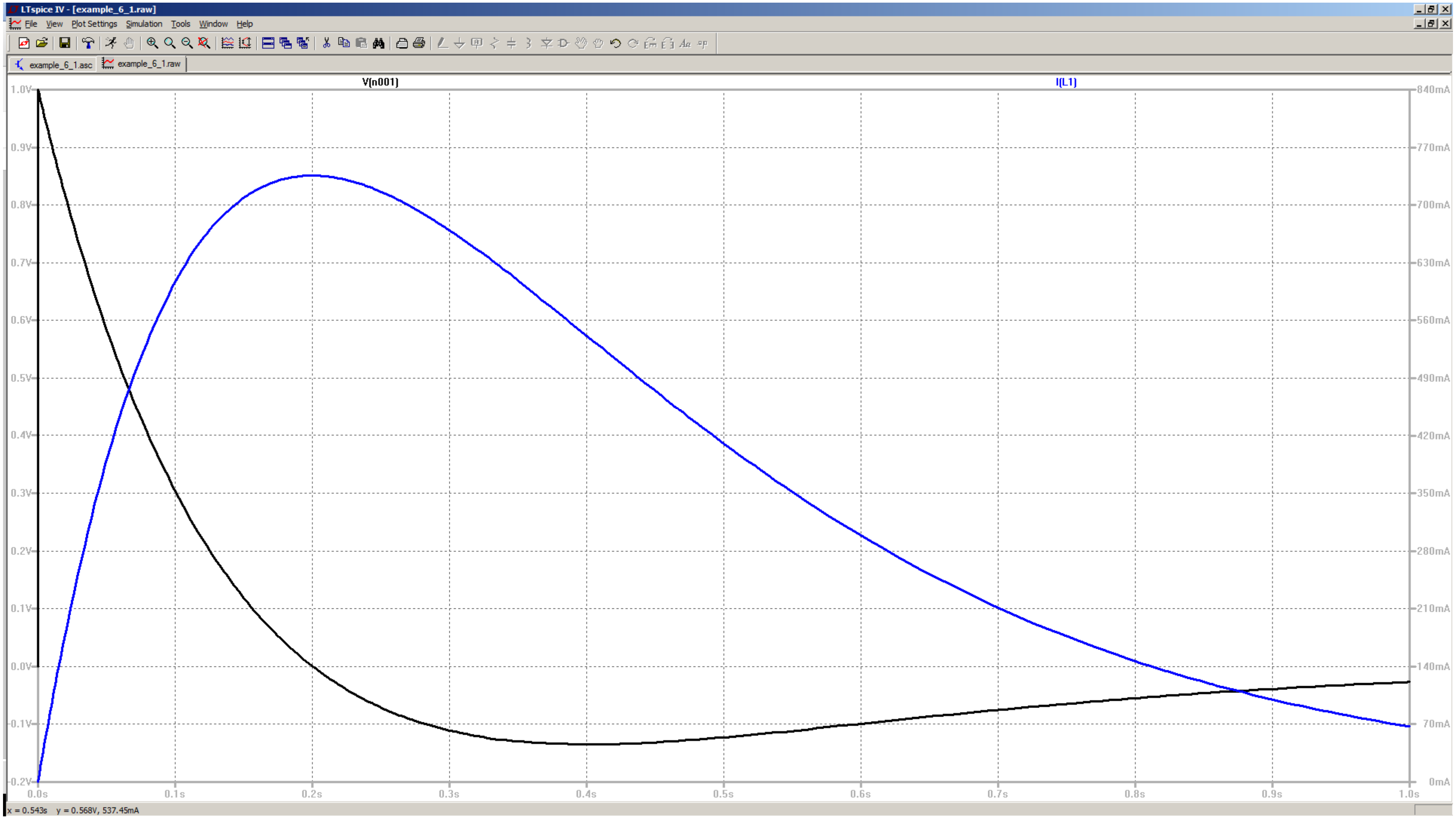
$v = L \frac{di}{dt}$ INDUCTOR

HOW DO YOU THINK INDUCTORS COMBINE IN SERIES? PARALLEL?

Example 6.1

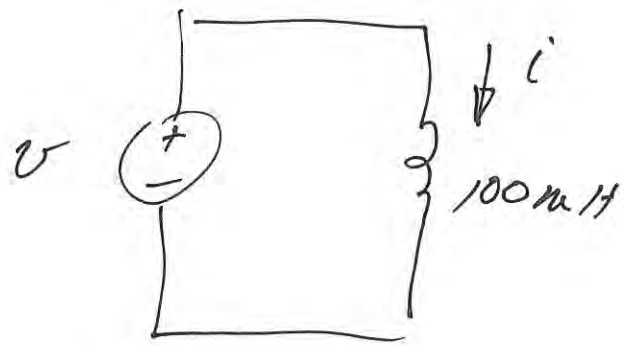


Implementation using behavioral current source



LTspice IV - [example_6_1.raw]
File View Plot Settings Simulation Tools Window Help
example_6_1.asc
example_6_1.raw
V(n001)
I(L1)
1.0V
0.9V
0.8V
0.7V
0.6V
0.5V
0.4V
0.3V
0.2V
0.1V
0.0V
-0.1V
-0.2V
840mA
770mA
700mA
630mA
560mA
490mA
420mA
350mA
280mA
210mA
140mA
70mA
0mA
0.0s
0.1s
0.2s
0.3s
0.4s
0.5s
0.6s
0.7s
0.8s
0.9s
1.0s
x = 0.543s y = 0.568V, 537.45mA

EXAMPLE 6.2



$$v(t) = 20te^{-10t}\ V$$

$$t \geq 0$$

WANT $i(t)$

$$i(t) = \frac{1}{L}\int_0^t v(x)\,dx$$

$$= \frac{1}{0.1}\int_0^t 20xe^{-10x}\,dx$$

BY PARTS: $u = x$, $dv = e^{-10x}dx$

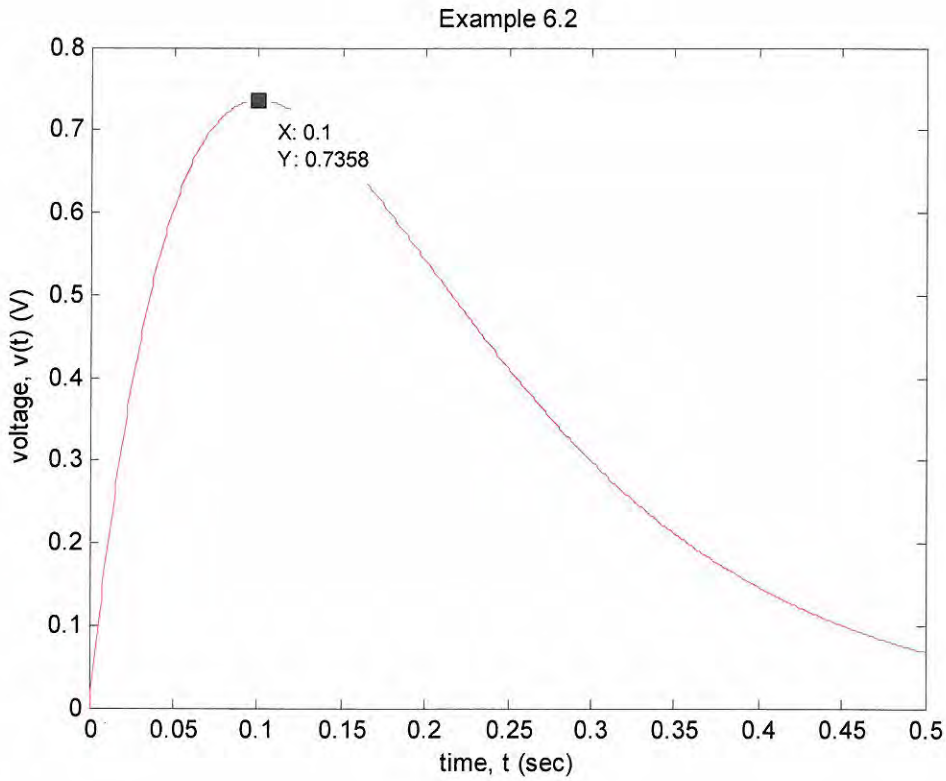
$du = dx$, $v = -\frac{1}{10}e^{-10x}$

$$i(t) = \frac{20}{0.1}\left\{ -\frac{x}{10}e^{-10x}\Big|_0^t + \frac{1}{10}\int_0^t e^{-10x}dx \right\}$$

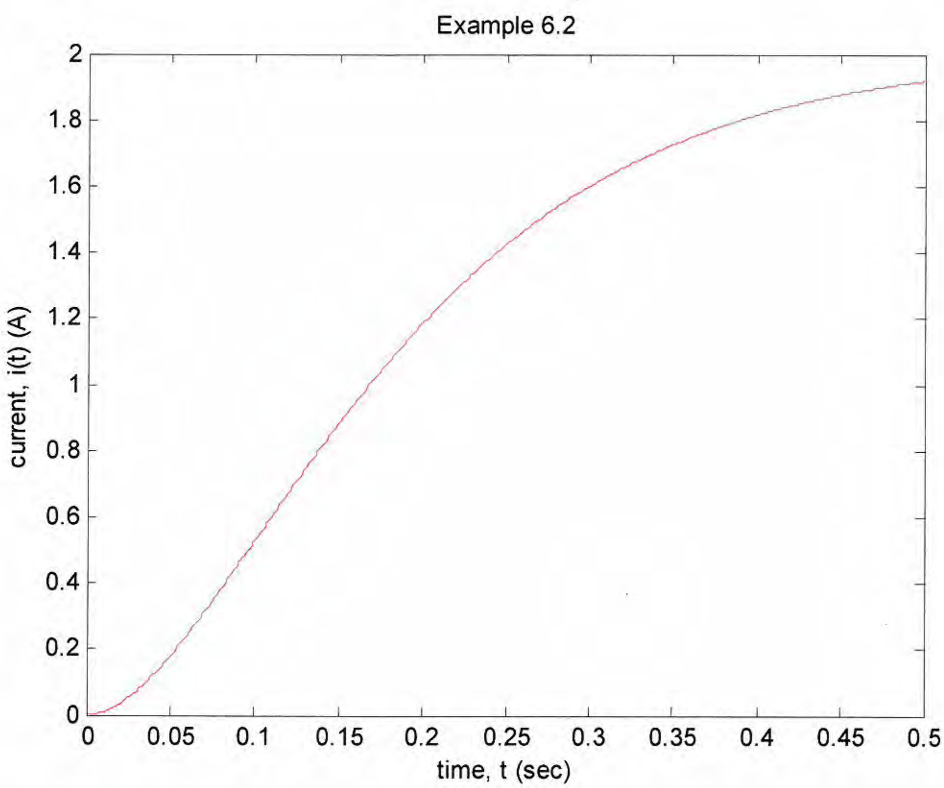
$$= \frac{20}{0.1}\left\{ -\frac{t}{10}e^{-10t} + \frac{1}{10}\left[-\frac{1}{10}e^{-10x}\Big|_0^t \right]\right\}$$

$$= \frac{20}{0.1}\left\{ -\frac{t}{10}e^{-10t} - \frac{1}{100}\left(e^{-10t} - 1\right) \right\}$$

$$i(t) = 2(1 - 10te^{-10t} - e^{-10t})A \; ; \; t \geq 0$$



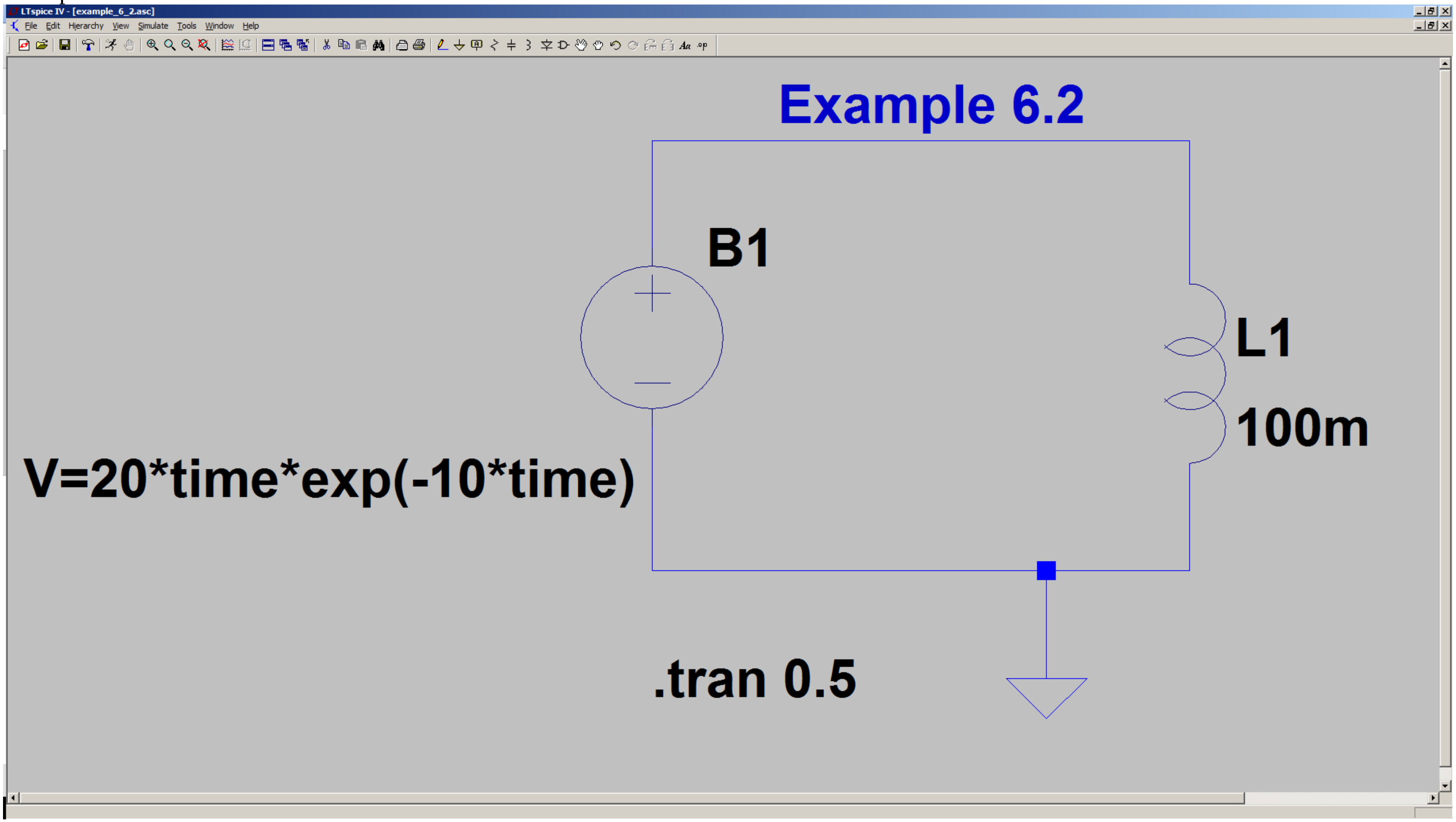
Example 6.2
X: 0.1
Y: 0.7358
voltage, v(t) (V)
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5
time, t (sec)

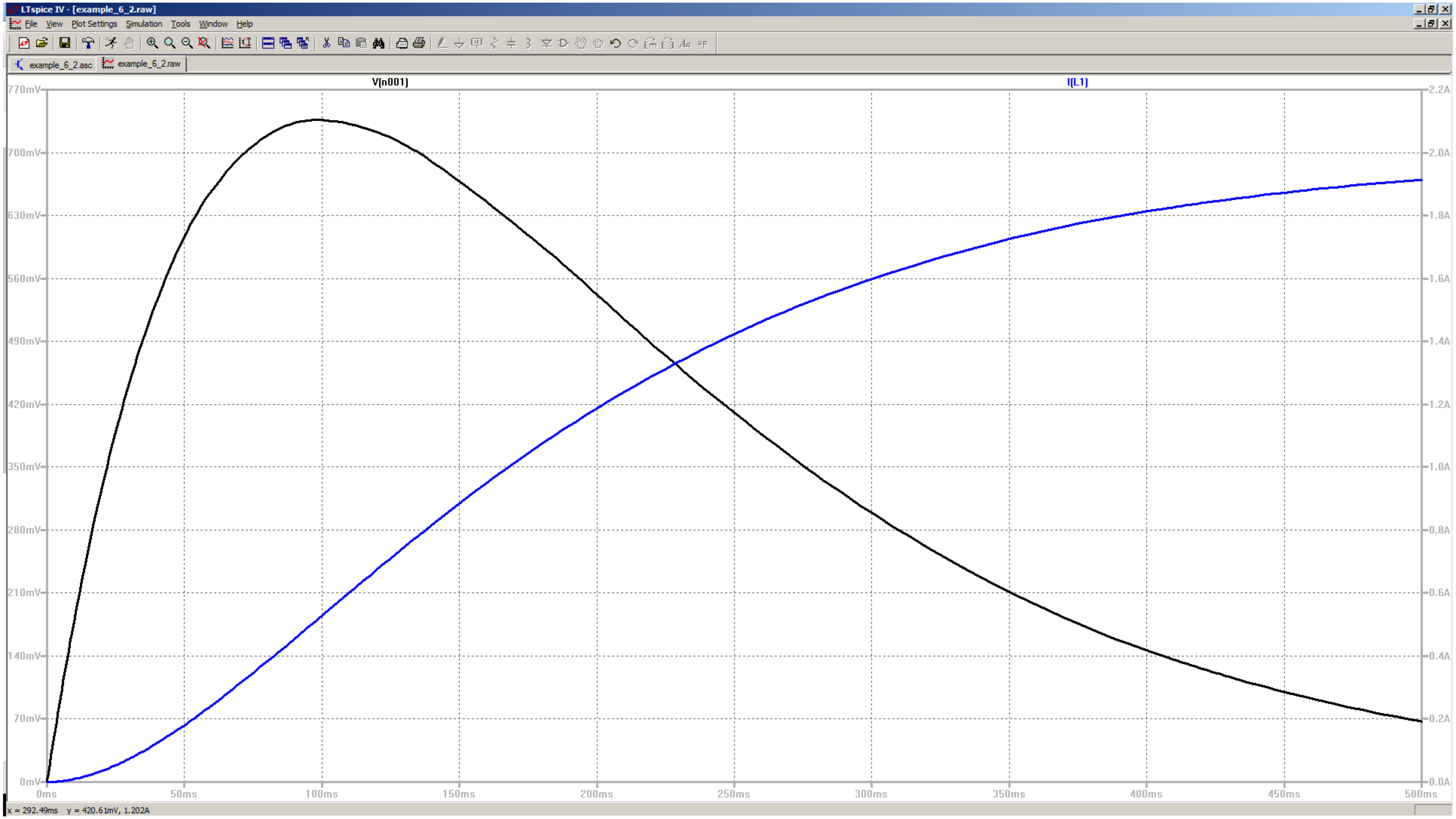


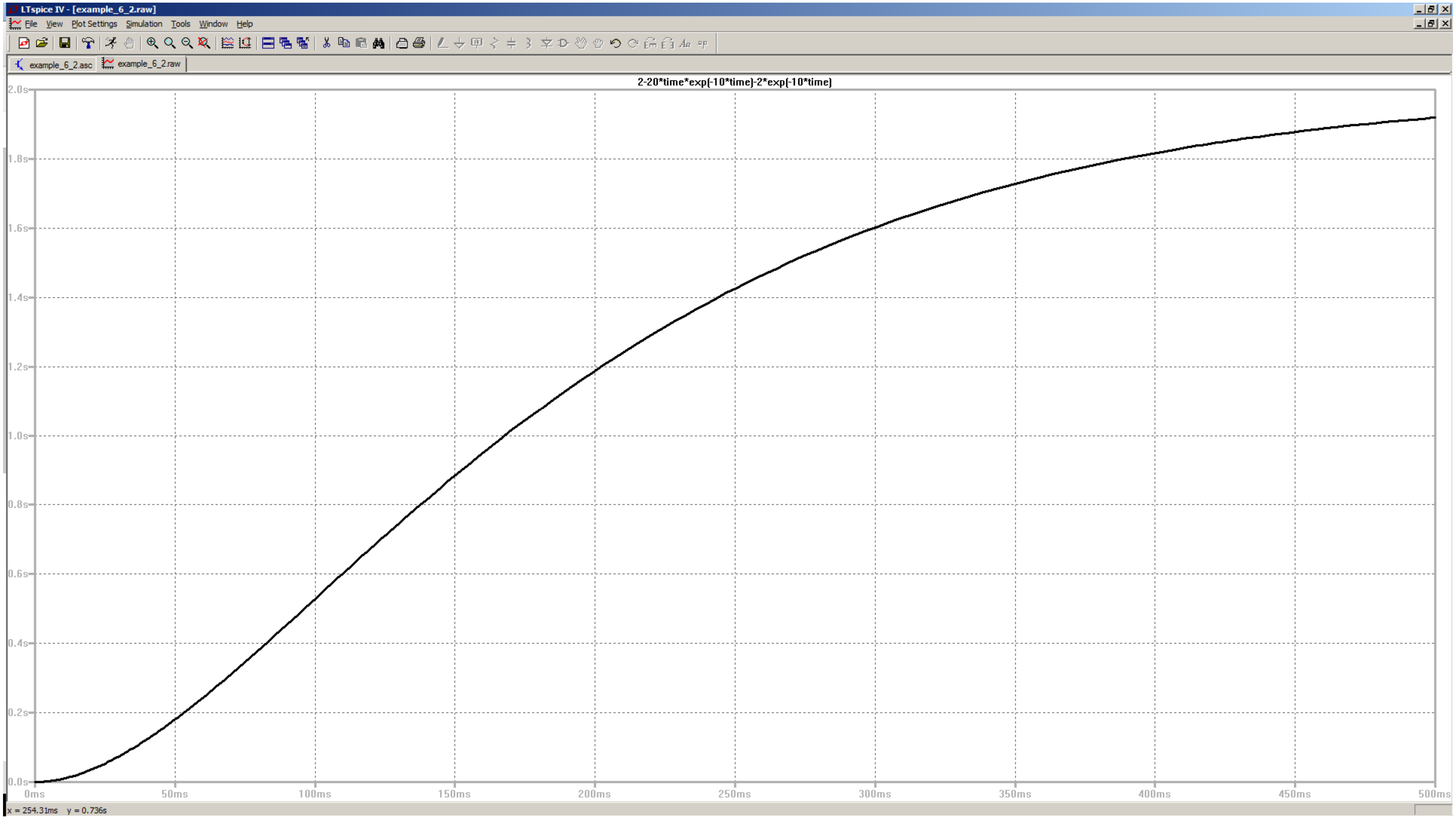
Example 6.2
current, i(t) (A)
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5
time, t (sec)

```
% example_6_2
% Nilsson & Riedel, 10th ed
% 01/22/15 D D Duncan
dt = 0.001;
t = 0:dt:0.5;% time in seconds
v = 20*t.*exp(-10*t);% voltage in volts
i = 2*(1 - exp(-10*t) - 10*t.*exp(-10*t));% current in amperes
figure(1);plot(t,v,'r-');
title('Example 6.2')
xlabel('time, t (sec)');ylabel('voltage, v(t) (V)')
figure(2);plot(t,i,'r-');
title('Example 6.2')
xlabel('time, t (sec)');ylabel('current, i(t) (A)')
```

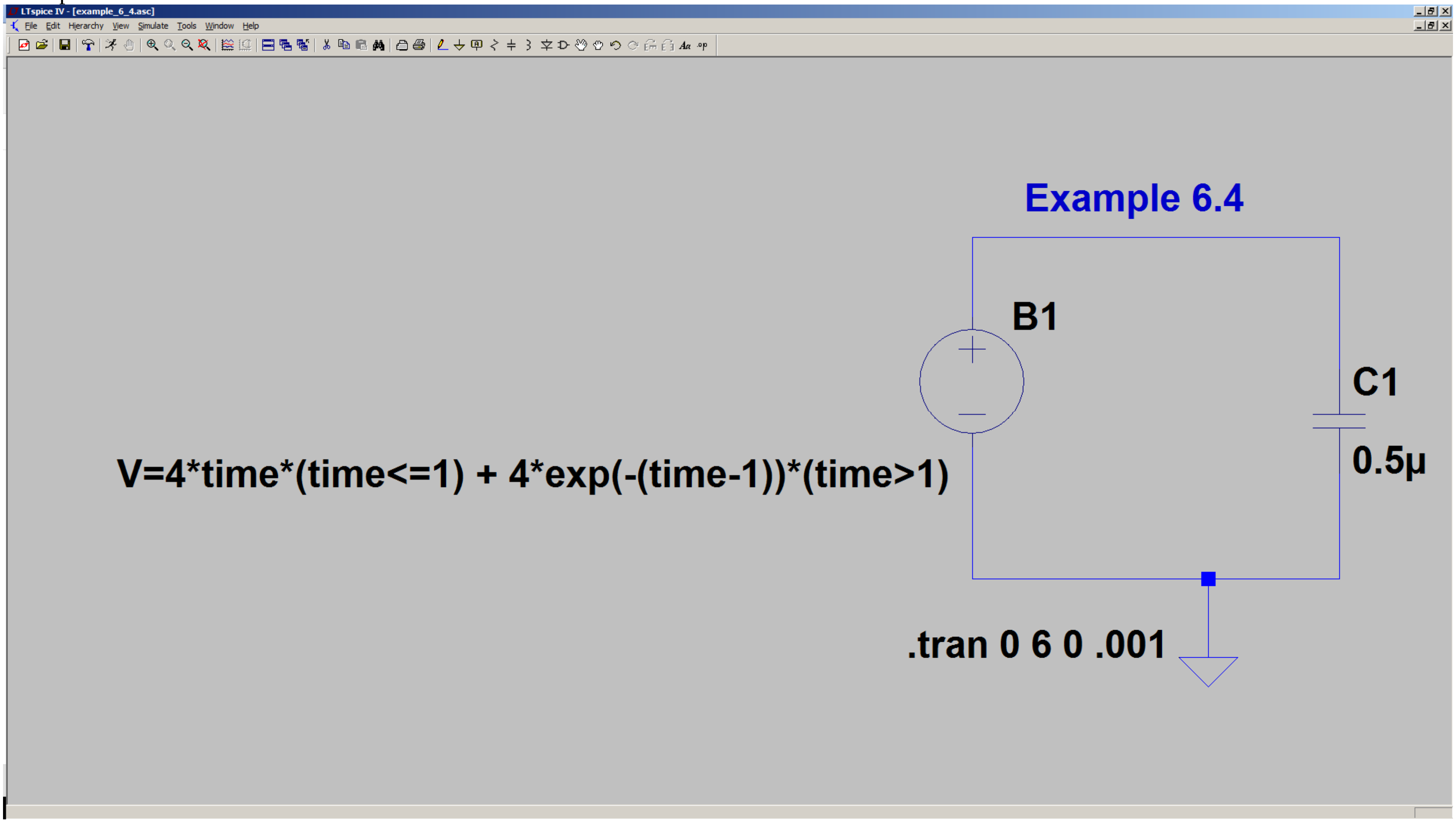
Example 6.2


LTspice IV - [example_6_2.asc]
File Edit Hierarchy View Simulate Tools Window Help
Example 6.2
B1
V=20*time*exp(-10*time)
L1
100m
.tran 0.5

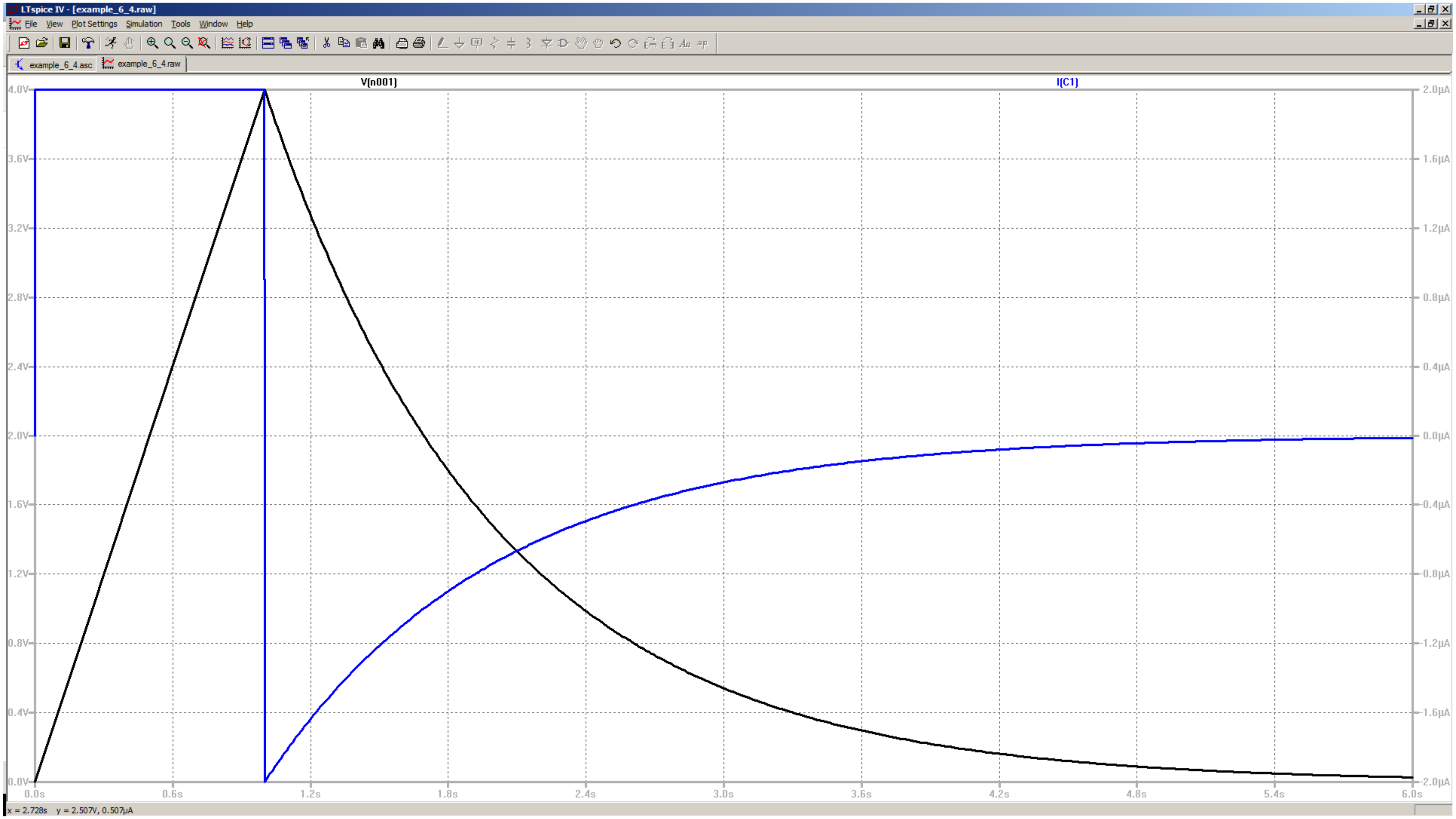

LTspice IV - [example_6_2.raw]
File View Plot Settings Simulation Tools Window Help
example_6_2.asc
example_6_2.raw
V(n001)
I(L1)
770mV
700mV
630mV
560mV
490mV
420mV
350mV
280mV
210mV
140mV
70mV
0mV
2.2A
2.0A
1.8A
1.6A
1.4A
1.2A
1.0A
0.8A
0.6A
0.4A
0.2A
0.0A
0ms
50ms
100ms
150ms
200ms
250ms
300ms
350ms
400ms
450ms
500ms
x = 292.49ms y = 420.61mV, 1.202A


LTspice IV - [example_6_2.raw]
File View Plot Settings Simulation Tools Window Help
example_6_2.asc
example_6_2.raw
2-20*time*exp(-10*time)-2*exp(-10*time)
2.0s
1.8s
1.6s
1.4s
1.2s
1.0s
0.8s
0.6s
0.4s
0.2s
0.0s
0ms
50ms
100ms
150ms
200ms
250ms
300ms
350ms
400ms
450ms
500ms
x = 254.31ms y = 0.736s

Example 6.4



More complex behavioral voltage source


LTspice IV - [example_6_4.raw]
File View Plot Settings Simulation Tools Window Help
example_6_4.asc
example_6_4.raw
V(n001)
I(C1)
4.0V
3.6V
3.2V
2.8V
2.4V
2.0V
1.6V
1.2V
0.8V
0.4V
0.0V
2.0µA
1.6µA
1.2µA
0.8µA
0.4µA
0.0µA
-0.4µA
-0.8µA
-1.2µA
-1.6µA
-2.0µA
0.0s
0.6s
1.2s
1.8s
2.4s
3.0s
3.6s
4.2s
4.8s
5.4s
6.0s
x = 2.728s y = 2.507V, 0.507µA